Rate change calculations

1. Introduction

This appendix complements Section 19.4.2 of the book, which only briefly touches on rate change calculations in the context of the pricing control cycle.

The rate change is a measure that tries to capture the extent to which an insurer is charging more (or less) to cover exactly the same risk from one year to the other. You can speak of rate change of an insurance contract, or a portion of an insurance contract (e.g., a layer of (re)insurance). You can also speak of the average rate change of a portfolio, or a sub-portfolio.

Some notes on monitoring rate changes (see Farr, Subasinghe, et al (2014)):

- An effective rate monitoring process is important to the success of an insurer and is an essential component of the pricing control cycle (see also Section 19.4.2)
- Rate changes can be calculated by the underwriters based on expert judgment, or by actuaries based on a pricing model, or by a mixture of the two (with cross-review/sign-off)
- Rate change calculations may focus on a single number, or may demand a breakdown of the different contributions (exposure, T&C, inflation, etc.)
- Very different results are obtained depending on the method used (e.g. whether a pricing model is used, or whether the past claims experience is factored in explicitly)

1.1 Terminology and notation

Symbol	Name	Description
e, e'	Exposure	Exposure at year $n-1$ and n. This can be a simple measure of exposure such as sum insured (property) or vehicle years (motor), or a more complex measure such as a property schedule (with detailed information on different locations/buildings) or a detailed fleet breakdown. In the latter case, the effect of assessing the impact of exposure on price involves not just scaling but recalculating the premium with the new schedule.
i, i'	Loss index	Loss index at year $n-1$ and n (if not incorporated in the differences in the rating models m, m')
c, c'	Cover	Cover at year $n-1$ and n. This is an umbrella term for all in relevant parameters defining the policy cover, e.g. deductibles, limits, aggregate deductibles/limits. Some of the parameters, e.g. local deductibles, might already be captured in the exposure definition and may therefore be excluded. The participation share is also excluded and captured by a separate parameter. Terms and conditions may be included here unless they are treated separately.

s, s'	Share	The participation share at year $n-1$ and n for a given contract of part of a contract, e.g. a layer of (re)insurance – we need to keep this separate from the rest of the cover structure.
m, m'	Model	The rating models used at year $n-1$ and n to produce a technical premium along with the parameters that go with it (e.g. exposure curve parameters, expense ratio, cost of capital…). This may incorporate the effect of loss inflation where needed, e.g. for experience rating and for liability exposure rating, in which case it is not necessary to have a different parameter i for the loss index. If we are using a uniform exposure rating model for a whole portfolio, m will be the same for the whole portfolio. Note that whether m, m' are included as part of the rate change calculations also depend on whether we want to factor out model revisions due to changes in historical experience (e.g. a recent increase in claims activity).
$\mathbf{TP}(e, i, c, s, m)$	Technical premium	This is the premium (also called "office premium") that the insurance company estimates to be adequate to cover expected losses, expenses/costs, and the desired level of profit. The technical premium is calculated for a given level of exposure e, inflation index i, cover (e.g. layer definition) c, co-insurance share s, pricing model m, and whatever other variable relevant to the situation at hand. This should ideally be purely technical (unaffected by commercial/market considerations) and is in general different from what the company actually charges.
$\mathbf{BP}, \mathbf{BP}'$	Bound premium	This is the premium actually charged (at year $n-1$ and n) by the company for a given contract, at the insurer's participation share.
$\mathbf{BP}_{\mathbf{asif}}$	As-if bound premium	The bound premium that would have been charged in year $n-1$ if the exposure, loss index, cover, share and model were as in year n.
$\mathbf{PAI}, \mathbf{PAI}'$	Premium adequacy index	This is an account-specific index that captures the profitability of the account (at year $n-1$ and n). It is defined as the bound premium divided by the technical premium.

2. Actuarial definition of rate change

The standard actuarial definition of rate change relies on having a sound pricing model, which allows to calculate the expected losses and the technical premium for a number of different scenarios.

2.2 Account rate change index and rate change ratio for a single contract (or part of a contract)

To begin with, let us start with defining the rate change for the smallest contractual unit (e.g. an employers' liability policy, or a layer of (re)insurance).

The rate change ratio can then be defined as the ratio between the bound premium at year n, $\mathbf{BP}'$**,** and the "as-if" bound premium at year $n-1$, $\mathbf{BP}_{\mathbf{asif}}$**.** This is the premium that would have been charged at year $n-1$ if the various parameters of the contract (e.g. exposure, cover, loss index…) had been the ones relevant for year n.

$$\rho_R := \frac{\mathbf{BP}'}{\mathbf{BP}_{\mathbf{asif}}} \tag{1.1}$$

$\mathbf{BP_{asif}}$ can be formally defined in terms of the change in the technical premium:

$$\mathbf{BP_{asif}} := \frac{\mathbf{TP}(e', i', c', s', m')}{\mathbf{TP}(e, i, c, s, m)} \times \mathbf{BP} \tag{1.2}$$

(See Section 1.1 for the definition of the symbols.)

The percentage rate change is then given by

$$\%\delta R = \rho_R - 1 = \frac{\mathbf{BP}'}{\mathbf{BP_{asif}}} - 1 \tag{1.3}$$

Finally, the change in absolute monetary amount is given by

$$\delta R = \mathbf{BP}' - \mathbf{BP_{asif}} = \%\delta R \times \mathbf{BP_{asif}} \tag{1.4}$$

From now on, we are going to use (1.1), (1.3) and (1.4) as the definitions for the rate change in the case of a single contractual unit.

2.1.1 Other sources of change

The factors included in the technical premium are e, i, c, s, m. Obviously, other factors subject to change may contribute to the technical premium and might therefore be looked at when considering the rate change. An example is given by the terms and conditions. There is formally no difficulty in including this explicitly as part of the technical premium. However, it should be noted that the impact of some of these factors on the TP are not easy to assess and they may have to be quantified by using judgment. In the following, we will assume that these effects can be ignored, or that they are somehow incorporated within the exposure e or the cover definition c. The treatment below can be extended to the case with different terms and conditions (or other risk factors) without any formal difficulty.

2.2.2 Relationship to the premium adequacy index

The premium adequacy index (PAI) for a contract is a measure of profitability, which tries to capture the extent to which the actual premium (or *bound* premium) is adequate, using the technical premium as a benchmark. It can be defined as

$$\mathrm{PAI} := \frac{\mathbf{BP}}{\mathbf{TP}(e, i, c, s, m)} \tag{1.2}$$

The change in profitability can be expressed as the ratio between the premium adequacy calculated at two different points in time. This is in turn equal to the rate change ratio:

$$\frac{\mathbf{PAI'}}{\mathbf{PAI}} = \frac{\mathbf{TP}(e, i, c, s, m)}{\mathbf{TP}(e', i', c', s', m')} \times \frac{\mathbf{BP}'}{\mathbf{BP}} = \rho_R \tag{1.3}$$

3. Breaking down the premium change into its components

Many companies are interested not only in determining the rate change but also the impact of the various factors (exposure, model, share, rate change…) in explaining how one goes from $\mathbf{BP}$ to $\mathbf{BP}'$ for a particular contract unit. This "walk-through" depends in general on the order with which we apply the changes and will look something like this (we will consider that the effect of change inflation is incorporated in the model change):

$$\mathbf{BP} \xrightarrow{m\to m\prime} \mathbf{BP}^{[m]} \xrightarrow{e\to e\prime} \mathbf{BP}^{[me]} \xrightarrow{s\to s\prime} \mathbf{BP}^{[mes]} \xrightarrow{c\to c\prime} \mathbf{BP}^{[mesc]} \equiv \mathbf{BP}_{\text{asif}} \xrightarrow{R\to R\prime} \mathbf{BP}'$$

This walk-through can be best understood in terms of the ratio between P_n^B and P_{n-1}^B, and by assuming that the effect of the various changes on the technical premium is multiplicative:

$$\frac{\mathbf{BP}'}{\mathbf{BP}} = \frac{\mathbf{BP}_{\text{asif}}}{\mathbf{BP}}\frac{\mathbf{BP}'}{\mathbf{BP}_{\text{asif}}} = \frac{\mathbf{TP}(m',e,s,c)}{\mathbf{TP}(m,e,s,c)}\frac{\mathbf{TP}(m',e',s,c)}{\mathbf{TP}(m',e,s,c)}\frac{\mathbf{TP}(m',e',s',c)}{\mathbf{TP}(m',e',s,c)}\frac{\mathbf{TP}(m',e',s',c')}{\mathbf{TP}(m',e',s',c)}\,\rho_{\text{R}}$$

By noting that $\mathbf{TP}(m,e,s,c) = s \times \mathbf{TP}(m,e,1,c)$, where $\mathbf{TP}(m,e,1,c)$ is the technical premium at 100% share, this can be re-written as

$$\frac{\mathbf{BP}'}{\mathbf{BP}} = \frac{\mathbf{TP}(m',e,1,c)}{\mathbf{TP}(m,e,1,c)}\frac{\mathbf{TP}(m',e',1,c)}{\mathbf{TP}(m',e,1,c)}\frac{s'}{s}\frac{\mathbf{TP}(m',e',1,c')}{\mathbf{TP}(m',e',1,c)}\,\rho_{\text{R}}$$

This can be written in a more compact form by giving a name to the ratios in the formula:

$$\rho_m = \frac{\mathbf{TP}(m',e,1,c)}{\mathbf{TP}(m,e,1,c)}$$

$$\rho_e = \frac{\mathbf{TP}(m',e',1,c)}{\mathbf{TP}(m',e,1,c)}$$

$$\rho_{\text{s}} = \frac{s'}{s}$$

$$\rho_{\text{c}} = \frac{\mathbf{TP}(m',e',1,c')}{\mathbf{TP}(m',e',1,c)}$$

Hence, the walk through the different changes from P_{n-1}^B to P_n^B is simply:

$$\frac{\mathbf{BP}'}{\mathbf{BP}} = \rho_m \times \rho_e \times \rho_s \times \rho_{\text{c}} \times \rho_{\text{R}} \qquad (1.5)$$

2.1 Rate change as percentage and as a monetary amount

Equation (1.5) has all the information we need about the different contributions to the premium change. However, it is often more useful to express these contributions in terms of percentage changes and monetary amounts.

Percentage changes can be obtained simply by subtracting 1 to the ratios above and expressing these numbers as percentages:

$$\%\delta m = \rho_m - 1$$

$$\%\delta e = \rho_e - 1$$

$$\%\delta s = \rho_s - 1$$

$$\%\delta c = \rho_c - 1$$

As for the monetary changes, these depend on the order with which the contributions are applied at (to some extent, this is also true for the percentage changes and the ratios). If one applies the changes in this order: model --> exposure --> share --> cover, the monetary changes can be written as:

$$\delta m = (\rho_m - 1) \times \mathbf{BP}$$

$$\delta e = (\rho_e - 1) \times \rho_m \times \mathbf{BP}$$

$$\delta s = (\rho_s - 1) \times \rho_e \times \rho_m \times \mathbf{BP}$$

$$\delta c = (\rho_c - 1) \times \rho_s \times \rho_e \times \rho_m \times \mathbf{BP}$$

Where δm, δe, δs and δc are the changes (in absolute monetary amounts) due to changes in model, exposure, share and cover.

The walk-through (see Equation (1.5)) can then also be written down additively as:

$$\mathbf{BP}' - \mathbf{BP} = \delta m + \delta e + \delta s + \delta c + \delta R \qquad (1.6)$$

3. Rate change at programme and portfolio level

The rate change calculations seen in Section 2 are for a single contract unit or layer of (re)insurance. There are several ways in which the results for the individual layers can be combined at programme/portfolio level. These vary depending on how we calculate the average change across various dimensions (e.g. share) and whether we require layers to be mapped into each other from one year to the next.

Note that in order to calculate the rate change for the portfolio, we limit ourselves to the contracts that are included in both year $n-1$ and year n. Lapses and new contracts do however have an impact on profitability, of course, and therefore should be considered when calculating the premium adequacy index.

Also, if we were interested in calculating the rate change not for the portfolio but more in general for the market or for a given type of business, then lapsed and new contracts would be relevant to the calculations.

3.1 Combining the rate change of different contract units

In the following, we assume that corresponding contracts and contract units are mapped explicitly.

The overall bound premium of the portfolio can be written as

$$\delta \mathbf{BP}^{\wp} = \sum_{\text{all matching units } j} \delta \mathbf{BP}_j$$

$$\delta \mathbf{BP}'^{\wp} = \sum_{\text{all matching units } j} \delta \mathbf{BP}'_j$$

Where the sum is extended over all contracts (or contract units) that are explicitly mapped between year $n-1$ and year n. In other words, contracts or layers that are dropped or new from one year to the other are not included in the calculations.

The monetary changes across a programme (or portfolio) $\wp$ can be written as the sum of the changes for each mapping layer:

$$\delta m^{\wp} = \sum_{\text{all matching units} j} \delta m_j$$

$$\delta e^{\wp} = \sum_{\text{all matching units } j} \delta e_j$$

$$\delta s^{\wp} = \sum_{\text{all matching units } j} \delta s_j$$

$$\delta c^{\wp} = \sum_{\text{all matching units } j} \delta c_j$$

$$\delta R^{\wp} = \sum_{\text{all matching units } j} \delta R_j$$

The change percentages can then be calculated as

$$\%\delta m^{\wp} = \frac{\delta m^{\wp}}{\mathbf{BP}}$$

$$\%\delta e^{\wp} = \frac{\delta e^{\wp}}{\mathbf{BP} + \delta m^{\wp}}$$

$$\%\delta s^{\wp} = \frac{\delta s^{\wp}}{\mathbf{BP} + \delta m^{\wp} + \delta e^{\wp}}$$

$$\%\delta c^{\wp} = \frac{\delta c^{\wp}}{\mathbf{BP} + \delta m^{\wp} + \delta e^{\wp} + \delta s^{\wp}}$$

$$\%\delta R^{\wp} = \frac{\delta c^{\wp}}{\mathbf{BP} + \delta m^{\wp} + \delta e^{\wp} + \delta s^{\wp} + \delta c^{\wp}}$$

Finally, the portfolio change ratios can be calculated as:

$$\rho_m^{\wp} = \%\delta m^{\wp} + 1$$

$$\rho_e^{\wp} = \%\delta e^{\wp} + 1$$

$$\rho_s^{\wp} = \%\delta s^{\wp} + 1$$

$$\rho_c^{\wp} = \%\delta c^{\wp} + 1$$

$$\rho_R^{\wp} = \%\delta R^{\wp} + 1$$

A calculation example is shown below.

CALCULATION EXAMPLE

Year n-1

n-1 (expiring)	
Exposure index	100
Inflation index	100
Overall TP@100%	150
Overall BP@100%	120
Overal TP@insurer's share	12.5
Overall BP@insurer's share	10.5
Average share (TP-weighted)	8.3%
Average share (BP-weighted)	8.8%
Premium adequacy index (PAI)	84.0%

Layer name	*Include? (1=Yes)*	*Description*	*Exposure index*	*TP @ 100%*	*BP @ 100%*	*Insurer's share*	*TP @ insurer's share*	*BP @ insurer's share*	*PAI*
Layer 1	1	20 xs 5	100	100	90	10%	10.0	9.0	90.0%
Layer 2	1	15 xs 25	100	50	30	5%	2.5	1.5	60.0%

Year n

n (renewal)	
Exposure index	110
Inflation index	100
Overall TP@100%	150
Overall BP@100%	90
Overal TP@insurer's share	10.5
Overall BP@insurer's share	8.1
Average share (TP-weighted)	7.0%
Average share (BP-weighted)	9.0%
Premium adequacy index (PAI)	77.1%

Layer name	*Include? (1=Yes)*	*Description*	*Exposure index*	*TP @ 100%*	*BP @ 100%*	*Insurer's share*	*TP @ insurer's share*	*BP @ insurer's share*	*PAI*
Layer 1'	1	20 xs 5	110	100	80	10%	10.0	8.0	80.0%
Layer 2'	1	15 xs 25	110	50	10	1%	0.5	0.1	20.0%

OVERALL WATERFALL CHART (WITH MAPPING)								
	BP(n-1)	*δExposure*	*δShare*	*δLayer*	*δInflation*	*BP(n-1,as-if)*	*δRate*	*BP(n)*
USDm	10.50	1.05	-1.32	-0.93	0.00	9.30	-1.20	8.10
%		*10.0%*	***-11.4%***	*-9.1%*	*0.0%*		***-12.9%***	

3.2 Limitations of this approach

The main limitations of this method are that:

(a) The portfolio-level rate change is limited to contracts (and actually, contract units such as layers of (re)insurance) that were in the portfolio in both year $n-1$ and year n. New business does not contribute to the rate change, and neither does the business that is not renewed. This may be desirable or not – for example, it does not help us to understand what the rate change has been not for the portfolio per se, but more in general for the type of business that we are writing.

(b) When a programme has more than one layer, it is necessary to map the layers at time $n-1$ and n. This causes several problems:

  i. Incorrect mapping may lead to errors in the calculation of the programme rate change.
  ii. Layers that are split or merged require special care and specific rules.
  iii. Underwriters may manipulate the rate change calculations by applying judgment as to which layer correspond to each layer (when the definition is not identical).
  iv. The process of mapping is time consuming and requires a non-trivial user interface.

The problems discussed in (b) can be mitigated by an algorithm that maps the layers at year $n-1$ and n automatically.

4. References

Bodoff, N.M. (2008), Measuring the rate change of a non-static book of property and casualty insurance business, Society of Actuaries

Farr, D., Subasinghe, H. et al (2014), Marine and energy pricing, GIRO Proceedings, Institute and Faculty of Actuaries