Introduction to commercial property pricing

1. Introduction

This appendix complements Chapter 23, which contains various examples of specialist pricing. It also adds to the treatment of property pricing in Chapter 21 as an example of exposure rating. The treatment in that chapter was mainly about reinsurance property pricing, and builds up on the comments on the specificities of commercial property insurance made in that chapter.

The typical approach to commercial property insurance is through exposure rating. Claims data is sometimes available, but that will for the most part be made of "attritional" (i.e., small) losses, with the losses that involve the damage to a significant proportion of a property are typically under- or over-represented, making an experience rating approach shaky.

2. Main types of cover

Building – Property damage

This insures the physical building (an office, a manufacturing plant, a refinery, an oil platform… -- in shorthand, a "location") against a number of named perils or more in general for "All risks" (with named exclusions). The cover is typically for all losses above a local deductible (i.e. a deductible specific to each location), given as a monetary amount, and up to the total insured value for property damage.

In practice, the underwriter will work on the basis of the maximum possible loss (MPL), which is the largest loss that can happen when everything that can go wrong goes wrong. Typically, the MPL is lower than the TIV, although there are pathological cases where this is not the case (e.g. if neighbouring sites are affected).

Building – Business interruption

This insures against the loss of profit/earnings consequential to a property damage (it is also called "consequential loss").

It is normally provided in excess of a local deductible expressed in number of days, or (less commonly) in monetary amount (typically, the deductible is a monetary amount when there is PD/BI combined deductible), and up to a maximum number of days called the "indemnity period" (IP). The indemnity period may start from the loss date or from after the deductible has been exhausted – that depends on the specifics of the contract.

BI cover can be sold under different bases, the most common of which are loss of profits and gross earnings.

Loss of profits

This is the most common basis outside of the US. It compensates the insured for any loss of profit over a timeframe called "indemnity period". More specifically, it compensates the insured for "gross profit", which is defined as[1]

$$\text{Gross Profit} = \text{Revenues} - \text{Variable costs}$$

where the variable costs are the ones that are incurred when the location is fully operational. The idea is that the insured should be covered for the lost revenues, but not for those costs that wouldn't anyway be incurred during a production halt.

Reasonable costs incurred to limit the impact of the interruption (increased cost of working, ICOW) are also normally covered.

Gross earnings

This is most common within the US and the cover it gives rise to is typically called "business income" (rather than business interruption/consequential loss). Unlike loss of profits, this has no fixed indemnity period as a reference. The cover is instead offered for the period of reinstatement (i.e. the time it takes for the property to be rebuilt or repaired and made functional again) plus possibly an extended indemnity period (EIP)). Notice that the insurer may limit the cover for the time it takes to rebuild "with due diligence and dispatch" (see Greaves & Suppiah, 2013).

This type of cover has obviously more flexibility than *loss of profits* with regard to the indemnity period, as the risk of running out of cover is reduced. On the flip side, gross earnings cover terminates abruptly once the reinstatement period or the extended indemnity period is finished, regardless of whether business has resumed as usual or not.

The loss is calculated with a similar principle to loss of profits, but with a slight difference in the calculation method[2], in the sense that the actual loss (gross earnings) are calculated starting from the bottom (net profit) and adding fixed costs (standing charges) to obtain the final loss:

$$\text{Gross Earnings} = \text{Net profit} + \text{Fixed costs}$$

Gross earnings also allow for the recovery of extra costs to keep the business running. This has the same function as ICOW for loss of profits and is called "extra expenses" (EE) in this context.

Machinery breakdown – Property Damage

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[1] This can also be written as $\text{Gross Profit} = \text{Net profit} + \text{Fixed costs}$, where $\text{Net profit} = \text{Revenues} - \text{Costs}$ and $\text{Costs} = \text{Variable costs} + \text{Fixed costs}$.

[2] There are also some differences in the items included. For example, finished stock is not covered by gross earnings, because this only covers future earnings and not stock that has already been produced.

This covers the insured against the loss of equipment consequential to a loss triggered by an external event (e.g. a fire) – this can be written based on named perils or on "all risks" with exclusions.

Note that this is not a cover for breakdowns caused by internal malfunctioning of machinery – an external event must trigger the loss.

Machinery breakdown – Business interruption

This is analogous to building business interruption and covers the insured against losses consequential to property damage to the machinery.

Other cover

Other cover that may be purchased aside from the standard PD/BI cover for both building and machinery is:

Contingent business interruption (CBI). According to the IRMI, "Contingent business interruption insurance and contingent extra expense coverage is an extension to other insurance that reimburses lost profits and extra expenses resulting from an interruption of business at the premises of a customer or supplier." (Torpey, 2003).

In the same way that business interruption is triggered by physical damage of the insured's property, CBI is triggered by damage to a property that does not belong to the insured but on which the business of the insured is somehow dependent – a supplier, a customer or even a neighbouring business whose presence is beneficial to the insured's business.

3. Pricing components

Property risks used to cover F/L/Ex/A risks, *i.e.* **fire, lightning, explosion and aircraft collision**. Today, they tend to be "All risks", which includes natural catastrophes such as earthquake, flood, hurricanes, etc. Some of these nat cat perils are modelled by companies such as RMS or EQECat (depending on territory)

Other perils will have to be dealt with by adding an ad hoc allowance. Overall, the losses can be broken down into:

$$S_{all} = S_{non-NC} + S_{mod-NC} + S_{nonmod-NC}$$

where

- S_{all} = total losses
- S_{non-NC} = non-natcat losses
- S_{mod-NC}= modelled nat-cat losses
- $S_{nonmod-NC}$= non-modelled nat cat losses

Sections 3.1 and 3.2 will provide more details on how these contributions can be calculated.

3.1 Non-cat (F/L/Ex/A) losses

These are typically priced based on exposure rating, although the use of experience rating is possible, especially for the attritional component of the claims. When both experience rating and exposure rating are used, credibility weighting can be used to combine the two results together. Since experience rating is very much the same as for other lines of business, we'll focus here on exposure rating.

3.1.1 Ingredients to pricing

A pricing exercise requires certain inputs about the client's property, such as the value of the property and its location, and on the coverage (e.g. retention level, limit). Underwriting guidelines (whether set out explicitly in a document or incorporated into a pricing model) will then use such information to determine a suitable rate on value (ROV) and ultimately the premium to be charged.

3.11.1 Client-specific inputs

A **property schedule** will normally be provided. This is a list of properties with information on:

- type of occupancy (e.g. refinery, paper mill, supermarket, etc.)
- total insured value (TIV) – this is part of the contract and can be further split between PD vs BI and building vs machinery breakdown
- maximum possible loss (MPL) – this is typically estimated by the underwriter based on the information that they receive from the client or from risk engineers
- local deductible (LD) – this deductible is "local" in the sense that it depends in general on the individual location
- location details – such as latitude/longitude, construction material, etc.: this information is especially useful to assess catastrophe risk
- indemnity period (for BI) – this is the maximum number of days of no activity/reduced activity which will be covered under the BI policy
- unit size (for Machinery Breakdown) – this is used in some lines of business such as power to differentiate between properties based on the wattage of the power produced.

Cover details (e.g. attachment point, limit, co-insurance share) will also need to be provided for pricing purposes.

3.1.1.2 Portfolio-level guidelines

For each type of occupancy (e.g., Crude Processing – Refineries), the following will typically be specified:

- A table of **standard deductibles** (SD), i.e. the typical level of local deductible for a certain occupancy. It is normally expressed in monetary amount for PD and in days for BI.
- A table of **base rates**, i.e. the premium or expected losses @ a given standard deductible. Normally expressed in ‰ of the insured value.
- A table of **exposure curves parameters**. An exposure curve $G(d)$ is a curve that provides the percentage of losses retained on average for a given level of retention d expressed as a percentage of MPL or TIV (see Chapter 21), and that allows the underwriter to allocate the premium to a layer of (re)insurance. The simplest and most common curves are the Swiss Re "c" curves, which have a single parameter c and can be generalised to two-parameter curves called MBBEFD curves (Chapter

21). The curves may either be from the ground up or, more commonly, they might be provided in excess of the SD.
Unless otherwise specified, we will assume that the exposure curve $G(d)$ is defined in terms of the MPL, and that the likelihood of a loss above MPL is zero. We will also look at how we can account for the possibility of losses above MPL.

- A table of BI-specific parameters, e.g. the factor by which the base rate should be multiplied for changes to the indemnity period.
- A table of **rates vs local deductibles**, that spells out the impact on the rate of changing deductibles, e.g. of using a local deductible that is different from the standard one. These are necessary if the exposure curves are provided in excess of a standard deductible and need to be restated in terms of a generic deductible. If the exposure curve is given from the ground up, such table is implicitly incorporated in the exposure curve.
- Rules on how much premium or expected losses should be allocated between MPL and IV

All the parameters above can of course be updated with time but some of these (e.g. the parameters of the exposure curves) tend to remain quite constant over the years (e.g. the Swiss Re and Lloyd's curves are still in popular use despite dating back to the 60s.

3.1.1.3 Company-level information

Rules for premium breakdown into expected losses, expenses, profit, etc. in a "Cost+" fashion (see Chapter 19).

Since these rules depend critically on the financials of the company (including such information as the cost of capital) they may change regularly.

3.1.2 Pricing methodology

Using the ingredients specified in Section 3.1.1, it is possible to tackle various pricing-related problems.

3.1.2.1 Rate on value, expected losses and premium for the full value of the property

By "full value" we normally mean the whole value of the property (TIV) in excess of the local deductible. Since we have a table that gives us the standard deductible for each type of occupancy, finding the rate at the local deductible is straightforward. The actual method depends on whether the exposure curve used by the company is from the ground up or in excess of the standard deductible.

Using exposure curves in excess of a standard deductible

In this case, the rate on value can be simply obtained from the base rate using the rates vs local deductible table. This table can be something as simple as a table like this:

Local deductible	***Multiplication factor***
50% of standard deductible	$\times 1.5$
100% of standard deductible	$\times 1.0$
150% of standard deductible	$\times 0.8$

200% of standard deductible	$\times$ 0.6

The table above allows us to calculate the rate on value by multiplying the base rate by the relevant multiplication factor: e.g. if the standard deductible is £100,000 and the the base rate is 0.5‰, the rate on value for a deductible of £150,000 is going to be $0.5‰ \times 0.8 = 0.4‰$. Obviously, if the value of the local deductible sits between different rows in the table above some interpolation is necessary.

A rather crude approach would allow for the multiplication factor to depend on the actual monetary value of the standard deductible and on the MPL (a £1m deductible will of course have less impact on a £1bn property than on a £10m property!).

In general, what we need is something that can be formally described as a transformation matrix $\mathbb{T}(SD, LD, MPL)$ that gives the multiplication factor to go from the standard deductible to a specific local deductible for a property with a given MPL:

$$\text{Rate on value } (LD) = \text{BR} \times \mathbb{T}(SD, LD, MPL)$$

where BR is the base rate (on an expected losses basis or a premium basis). The transformation matrix can be given in the form of a table or as an algebraic relation, such as (see Lee & Frees, 2016):

$$\mathbb{T}(SD, LD, MPL) = \left(\frac{\theta + \frac{LD}{MPL}}{\theta + \frac{SD}{MPL}}\right)^{-\alpha+1}$$

$$\mathbb{T}(SD, LD, MPL) = 1 - \frac{\left((a+1) \times b^{\frac{LD}{MPL}}\right)}{a + b^{\frac{LD}{MPL}}} = \frac{a \times \left(1 - b^{\frac{LD}{MPL}}\right)}{a + b^{\frac{LD}{MPL}}}$$

This approach can be used for both PD and BI.

If BR is the premium base rate, the premium to be charged for the full value is simply

$$\text{Premium} = \text{ Rate on value}^{P} \ (LD) \times TIV$$

If BR is the expected losses base rate, the expected losses are given by

$$\text{Expected losses} = \text{ Rate on value}^{EL} \ (LD) \times TIV$$

and the premium is obtained as

$$\text{Premium} = \text{Expected losses} + \text{Loadings}$$

Using from-the-ground-up exposure curves

In this case, the rate on value can be obtained from the base rate using the properties of the exposure curves.

Let BR be the base rate (on an expected losses basis), and FGU_Rate the rate that would be applied if there were no deductible. Then (from the very definition of exposure curve – see Chapter 21):

$$BR^{EL} = \text{FGU\_Rate} \times \left(1 - G\left(\frac{SD}{MPL}\right)\right)$$

and

$$\text{Rate on value}^{EL}(\text{LD}) = \text{FGU\_Rate}^{EL} \times \left(1 - G\left(\frac{LD}{MPL}\right)\right)$$

Hence,

$$\text{Rate on value}^{EL}(\text{LD}) = BR^{EL} \times \frac{1 - G\left(\frac{LD}{MPL}\right)}{1 - G\left(\frac{SD}{MPL}\right)}$$

(We have assumed that no premium is allocated between MPL and TIV.)

The results above hold if the base rate and the rate on value are given on an expected losses basis. They also hold for the case where they are on a premium basis and the loadings can be assumed to be proportional to the expected losses. If a more complex Cost+ model is adopted, then the relationship between the ROV at LD and the base rate at SD may be distorted and stochastic simulation may be needed.

3.1.2.2 Expected losses and premium for a layer of (re)insurance

Again, this calculation proceeds differently depending on whether we have a from-the-ground-up exposure curve or our exposure curve is on top of a standard deductible.

Using exposure curves in excess of a standard deductible

The expected losses to a layer $L\ xs\ D$, with $L + D < MPL$ under a local deductible equal to LD can in this case be written as

$$E\big(S(D,L)\big) = BR^{EL} \times \mathbb{T}(SD, LD, MPL) \times \left(G\left(\frac{D+L}{MPL}\right) - G\left(\frac{D}{MPL}\right)\right) \times TIV$$

The premium can then be obtained by

$$\text{Premium} = E\big(S(D,L)\big) + \text{Loadings}$$

Note that the calculation of the loadings may require Monte Carlo simulation or similar techniques when it includes an allowance for volatility (see Section 3.1.2.3).

If the base *premium* rate BR^P is used, the premium is instead directly

$$\text{Premium} = BR^P \times \mathbb{T}(SD, LD, MPL) \times \left(G\left(\frac{D+L}{MPL}\right) - G\left(\frac{D}{MPL}\right)\right) \times TIV$$

Using from-the-ground-up exposure curves

The expected losses to a layer $L \; xs \; D$, with $L + D < MPL$ under a local deductible equal to LD can in this case be written as

$$E\big(S(D,L)\big) = \frac{BR^{EL}}{1 - G\left(\frac{SD}{MPL}\right)} \times \left(G\left(\frac{D+L+LD}{MPL}\right) - G\left(\frac{D+LD}{MPL}\right)\right) \times TIV$$

As before, the premium can then be obtained by

$$\text{Premium} = E\big(S(D,L)\big) + \text{Loadings}$$

If the base *premium* rate BR^P is used, the premium is instead directly given by

$$\text{Premium} = \frac{BR^{P}}{1 - G\left(\frac{SD}{MPL}\right)} \times \left(G\left(\frac{D+L+LD}{MPL}\right) - G\left(\frac{D+LD}{MPL}\right)\right) \times TIV$$

3.1.2.3 Loss distribution

The exposure curve approach can also be used for stochastic modelling, using a frequency/severity approach to produce not only an estimate of the expected losses for the full value or for a layer, but also an estimate of the volatility and of the full aggregate loss distribution.

In order to adopt this approach, we need first of all a frequency model. A commonly adopted frequency model is the Poisson distribution, which should normally be sufficient if systemic effects can be disregarded (see Chapter 14), which is a reasonable assumption in commercial property insurance. A negative binomial distribution (or another over-dispersed distribution) can be used for extra volatility. In the following, we will use a Poisson model:

$$N \sim \text{Poi}(\lambda)$$

where λ is the expected number of claims for that location (either from the ground up or above the local deductible).

As for the severity model, this is uniquely determined by the exposure curve $G(x)$, which is in one-to-one correspondence with the normalised severity distribution $F(x)$ by the following: relationship (see Chapter 21 for more details):

$$F(x) = \begin{cases} 1 - \frac{G'(x)}{G'(0)} & 0 \le x < 1 \\ 1 & x = 1 \end{cases}$$

where $x = \frac{X}{MPL}$ in the case where the exposure.

As usual, it is a good idea to look separately at the case where the exposure curve is defined in excess of a deductible and the case where it is from the ground up.

In both cases, the Poisson rate for the number of claims can be easily determined from the knowledge of the expected aggregate losses for a given location, $E(S)$, and an application of the usual equality,

$$E(S) = E(N) \times E(X) = \lambda \times E(X)$$

3.1.2.3.1 Using exposure curves in excess of the standard deductible

The expected severity in excess of the standard deductible is given by (see Chapter 21):

$$E(X) = MPL \times E(x) = \frac{MPL}{G'(0)}$$

(this is the same formula as for the expected ground-up severity, but in this context it describes the amount in excess of the deductible).

The approximation inherent to this method is that the exposure curve is valid for *any* level of the local deductible, and that the only thing that changes when changing the deductible is the frequency of losses.

The frequency in excess of the standard deductible is

$$\lambda_{[>SD]} = \frac{BR^{EL} \times TIV \times G'(0)}{MPL}$$

and the frequency in excess of a generic local deductible can be obtained by:

$$\lambda_{[>LD]} = \frac{BR^{EL} \times TIV \times G'(0)}{MPL} \times \mathbb{T}(SD, LD, MPL)$$

The simulation algorithm then goes as follows:

For $j = 1$ to N (N= number of scenarios, or simulations)

(a) For each scenario j sample a number n_j of losses from a Poisson distribution with rate $\lambda_{[>LD]}$

(b) For each of the n_j losses, sample a severity $X_1^j, X_2^j \dots X_{n_j}^j$ from $F(x)$. The actual FGU severity can then be obtained by adding LD to each loss: $X_1^j + LD, X_2^j + LD \dots X_{n_j}^j + LD$

(c) Calculate the aggregate loss for scenario j: $S_j = X_1^j + X_2^j \dots X_{n_j}^j + n_j \times LD$

(d) Sort the values of S_j in ascending order and calculate the relevant statistics (e.g. mean, standard deviation, percentiles)

The outcome of this simulation is an empirical aggregate loss distribution on a gross basis. If we are interested in losses for a specific policy structure (e.g. a layer of reinsurance) we can as usual (see Chapter 17) apply to each of the losses X_k^j generated by the process the relevant policy modifier, and then apply the relevant aggregate modifiers to the S_j.

3.1.2.3.2 Using from-the-ground-up exposure curves

The expected severity from the ground up is given by (see Chapter 21):

$$E(X) = MPL \times E(x) = \frac{MPL}{G'(0)}$$

The frequency from the ground up is then

$$\lambda = \frac{BR^{EL} \times TIV \times G'(0)}{\left(1 - G\left(\frac{SD}{MPL}\right)\right) \times MPL}$$

The frequency above the local deductible is then

$$\lambda_{[>LD]} = \frac{BR^{EL} \times TIV \times G'(0) \times \left(1 - G\left(\frac{LD}{MPL}\right)\right)}{\left(1 - G\left(\frac{SD}{MPL}\right)\right) \times MPL}$$

In order to simulate losses above the local deductible we can proceed as follows.

For $j = 1$ to N (N= number of scenarios, or simulations)

(e) For each scenario j sample a number n_j of losses from a Poisson distribution with rate $\lambda_{[>LD]}$

(f) For each of the n_j losses, sample a severity $X_1^j, X_2^j \dots X_{n_j}^j$ from $F(x)$, conditional on x being larger than LD/MPL (this can be done by sampling from $F(x)$ and discarding the sample if $x < \frac{LD}{MPL}$)

(g) Calculate the aggregate loss for scenario j: $S_j = X_1^j + X_2^j \dots X_{n_j}^j$

(h) Sort the values of S_j in ascending order and calculate the relevant statistics (e.g. mean, standard deviation, percentiles)

The outcome of this simulation is an empirical aggregate loss distribution on a gross basis. If we are interested in losses for a specific policy structure (e.g. a layer of reinsurance) we can as usual (see Chapter 17) apply to each of the losses X_k^j generated by the process the relevant policy modifier, and then apply the relevant aggregate modifiers to the S_j.

3.1.2.3.3 Simulation of PD and BI components

The process must be repeated for PD and BI. The severity of the PD and the BI component of each loss can be correlated by a copula in the usual way as explained, e.g., in Section 28.5.

The policy modifiers will need to be applied mindful of whether they apply to each component separately or to PD and BI separately (combined deductible).

3.1.2.3.4 Contracts with multiple locations

When a contract/account has more than one location, the overall loss distribution can be derived easily if we assume that the losses arising from the different locations are independent. In this case, if we have locations $l = 1,2 \ldots n_L$, we can calculate the aggregate losses S_j^l for each location and for each scenario. The aggregate loss for scenario j across all locations is then $S_j^{\text{all}} = \sum_{l=1}^{n_L} S_j^l$. The overall average frequency is $\lambda_{[>LD\,]}^{\text{all}} = \sum_{l=1}^{n_L} \lambda_{[>LD_l]}^l$, where the local deductible in general changes depending on location. Apart from this, the calculations proceed exactly in the same way as for the single-location case.

3.2 Natural catastrophe losses

3.2.1 Modelled nat cat losses

This can be carried out using proprietary models or, more commonly, using professional tools by vendors such as RMS, EQECat, AIR.

The inputs required by these models is the exact location of all relevant properties and the characteristics of each property. This information can then be fed into a catastrophe modelling system (specifically, to the vulnerability module – see Chapter 24) to produce an event loss table from which the annual aggregate losses and eventually a full frequency/severity model for cat losses (see Box 24.1) can be constructed

More information on how nat cat losses can be modelled can be found in Chapter 24.

3.2.2 Non-modelled nat cat losses

A simplified approach must be adopted for losses for which no professional cat model exists.

Since non-modelled nat cat losses should be part of the company's internal capital model, a simplified model will have been developed in this context (based on loss experience, a survey of expert opinion, etc.). The output of this component of the internal model can then be used for pricing purposes.

Alternatively, an ad-hoc statistical analysis of past loss experience across all non-modelled perils may provide the starting point for a simplified frequency/severity model of such losses. In the simplest of implementations, the non-modelled nat cat losses may be taken into consideration simply by adding a percentage amount to the non-cat expected losses.

3.3 Overall losses

In Section 3.1 and 3.2 we have seen how it is possible to calculate the expected losses and indeed the full distribution of the non-cat, modelled cat and non-modelled cat components. We are now in a position to calculate the statistical properties of the overall losses.

The expected losses (on a gross basis) are easy to calculate since they are the sum of the various contributions:

$$E(S_{all}) = E(S_{non-NC}) + E(S_{mod-NC}) + E(S_{nonmod-NC})$$

As for the loss distribution, this is also relatively straightforward if we can make the assumption that the three contributions above are statistically independent. This seems relatively uncontroversial as, say, fire risk and earthquakes are obviously uncorrelated (unless when fire is directly caused by an earthquake in which case it can be considered earthquake from a cover point of view). The only thing we need to be aware of is that if $S_{nonmod-NC}$ (Section 3.2.2) is modelled as a fixed percentage of the non-cat losses, this may create perfect correlation if we are not careful in the way we simulate the losses – a good way of avoiding this is to create a separate model in which the frequency is a frequency of the non-cat frequency but form which we can sample independently.

A good way of proceeding is as follows. Let us assume that we have three models for non-cat, modelled nat cat and non-modelled nat cat:

$$N_{NC} \sim \mathrm{Poi}(\lambda_{NC}), X_{NC} \sim F_{X_{NC}}$$

$$N_{MNC} \sim \mathrm{Poi}(\lambda_{MNC}), X_{MNC} \sim F_{X_{MNC}}$$

$$N_{NMNC} \sim \mathrm{Poi}(\lambda_{NMNC}), X_{NMNC} \sim F_{X_{NMNC}}$$

Generate a large number N of different scenarios

For each scenario j,

- Draw a number of losses from $N_{NC}, N_{MNC}, N_{NMNC}$, let's say $n_{NC}, n_{MNC}, n_{NMNC}$
- For each of the n_{NC} losses, draw a severity from $X_{NC} \sim F_{X_{NC}}$. Analogously for MNC and NMNC
- The aggregate losses for scenario j are the sum of all loss amounts sampled for the three different components (n_{NC}+n_{MNC}+n_{NMNC} losses overall)

The main statistics of the aggregate loss distribution can then be calculated in the usual way. Note that the losses from the different components are by construction independent[3].

[3] The only thing we need to be aware of is that if S_{NMNC} (Section 3.2.2) is modelled as a fixed percentage of the non-cat losses, this may create perfect correlation if we are not careful in the way we simulate the losses – a good way of avoiding this is to create a separate model in which the frequency is a frequency of the non-cat frequency but form which we can sample independently. In the case above, for example, we can have $N_{NC} \sim \mathrm{Poi}(\lambda_{NC})$, $X_{NC} \sim F_{X_{NC}}$ $N_{NMNC} \sim \mathrm{Poi}(\alpha\lambda_{NC}), X_{NMNC} \sim F_{X_{NC}}$.

4. Examples

4.1 Example 1

Consider a food manufacturing plant with the following characteristics:

- TIV (PD) = £200m
- TIV (BI) = £200m
- MPL (PD) = £130m
- MPL (BI) = £200m
- Local deductible (LD) for PD = £1m
- LD(BI) = 45d
- Indemnity period (IP) = 12 months (from the accident date)

The underwriting guidelines for food manufacturing plants are as follows:

- The standard deductible is £1m for PD and 30 days for BI.
- Assume no allowance for losses between MPL and TIV.
- The exposure curves in use are the Swiss Re curve with c=3.8 for PD, c=3.1 for BI. In both cases, these are defined above the standard deductible.
- The base rate = 0.7‰ (PD), 1‰ (BI)
- The premium breakdown is as follows: 80% to cover the expected losses, 20% to cover costs and profit.
- The company also has a deductible impact table as below. The company uses it for both PD and BI.

	LD/SD=50%	LD/SD=100%	LD/SD=150%	LD/SD=200%
MPL=£100m-	125%	100%	90%	85%
MPL=£150m	120%	100%	92%	87%
MPL=£200m+	115%	100%	93%	88%

Where “MPL=£100m-“ means an MPL of £100m or less, and “MPL=£200m+” means an MPL of £200m or more.

Calculate:

(1) The rate at the local deductible for both PD and BI

(2) The expected losses to the layer £90m xs £10m (all in excess of the local deductible)

Solution

(1)

PD rate @ LD
SD=\$1m, LD=\$500k → LD/SD=50%
The MPL (£130m) is between £100m and £150m. By linear interpolation,

$$\text{PD Rate@LD} \sim 122\% \times 0.7‰ = 0.854‰$$

BI rate @ LD
SD=30d, LD=45d → LD/SD=150%
Using the row for MPL=£200m+, we obtain

$$\text{BI Rate@LD} = 93.0\% \times 1‰ = 0.93‰$$

(2)

PD expected losses

$$EL_{PD} = \left(G\left(c; \frac{L+D}{MPL_{PD}}\right) - G\left(c; \frac{D}{MPL_{PD}}\right)\right) \times LR \times \text{PD\_Rate@}LD \times TIV_{PD} =$$
$$= \left(G\left(3.8; \frac{100}{130}\right) - G\left(3.8; \frac{10}{130}\right)\right) \times 80\% \times 0.854‰ \times £200m = £63.5k$$

BI expected losses

$$EL_{BI} = \left(G\left(c; \frac{L+D}{MPL_{BI}}\right) - G\left(c; \frac{D}{MPL_{BI}}\right)\right) \times LR \times \text{BI\_Rate@}LD \times TIV_{BI} =$$
$$= \left(G\left(3.1; \frac{100}{130}\right) - G\left(3.1; \frac{10}{130}\right)\right) \times 80\% \times 0.930‰ \times £300m = £120.7k$$

4.2 Example 2

Consider the same food manufacturing plant as in Example 1.

The underwriting guidelines for food manufacturing plants are also as before, with the following exceptions:

- The exposure curves in use are the Swiss Re curve with c=3.8 for PD, c=3.1 for BI. In both cases, these are defined *from the ground up*.
- Since the curves are from the ground up, there is no need for a deductible impact table

As for Example 1, calculate:
(1) The rate at the local deductible for both PD and BI
(2) The expected losses to the layer £90m xs £10m (all in excess of the local deductible)

Solution

(1)

$$\text{PD\_Rate@LD} = \frac{1 - G(3.8; LD/MPL_{PD})}{1 - G(3.8; SD/MPL_{PD})} \times \text{Base\_Rate} = \frac{1 - 0.080}{1 - 0.137} \times 0.7‰ = 0.746‰$$

$$\text{BI\_Rate@LD} = \frac{1 - G(3.1; LD/MPL_{BI})}{1 - G(3.1;\ SD/MPL_{BI})} \times \text{Base\_Rate} = \frac{1 - 0.462}{1 - 0.383} \times 1‰ = 0.873‰$$

$$\text{PD\_Rate@0} = \frac{1}{1 - G(3.8; SD/MPL_{PD})} \times \text{Base\_Rate} = \frac{1}{1 - 0.137} \times 0.7‰ = 0.811‰$$

$$\text{BI\_Rate@0} = \frac{1}{1 - G(3.1;\ SD/MPL_{BI})} \times \text{Base\_Rate} = \frac{1}{1 - 0.383} \times 1‰ = 1.622‰$$

(2)

PD expected losses

$$\begin{aligned} EL_{PD} &= \left(G\left(c; \frac{L + D + LD_{PD}}{MPL_{PD}}\right) - G\left(c; \frac{D + LD_{PD}}{MPL_{PD}}\right)\right) \times LR \times \text{PD\_Rate@0} \times TIV_{PD} = \\ &= \left(G\left(3.8; \frac{100.5}{130}\right) - G\left(3.8; \frac{10.5}{130}\right)\right) \times 80\% \times 0.811‰ \times £200m = £59.1k \end{aligned}$$

BI expected losses

First of all we need to translate LD into a monetary amount. We do that by assuming that the monetary amount for business interruption is proportional to the number of days, and that it is distributed uniformly over the indemnity period of 12 months:

$$LD_{BI} = \frac{45}{365} \times £300m = £36.7m$$

$$\begin{aligned} EL_{BI} &= \left(G\left(c; \frac{L + D + LD_{BI}}{MPL_{BI}}\right) - G\left(c; \frac{D + LD_{BI}}{MPL_{BI}}\right)\right) \times LR \times \text{BI\_Rate@0} \times TIV_{BI} = \\ &= \left(G\left(3.1; \frac{136.7}{300}\right) - G\left(3.1; \frac{46.7}{300}\right)\right) \times 80\% \times 1.622‰ \times £300m = £97.9k \end{aligned}$$

5. References

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